Ballast Flight Risk Assessment Based on Reliability Theory

Shuai Shao [1,2], Guoqing Jing [2*], Hao Yin [3]

1 Beijing's Key Laboratory of Structural Wind Engineering and Urban Wind Environment, Beijing 100044, China.
2 *School of Civil Engineering*, Beijing Jiaotong University, Beijing 100044, China.
3 Water Resources and Civil Engineering School, China Agricultural University, Beijing 100083, China.

Abstract **- With the development of High Speed Railway, an apparent rail and train damage phenomena that occurs at the train-track interface associated with aerodynamic is termed "ballast flight". The ballast particles might become airborne during train passage when the aerodynamic and mechanical forces exceed the ballast particle's gravity and lead to damage to the entire rail and rolling stocks which results in a major maintenance cost and safety concern with ballasted track. The core elements including aerodynamic conditions, track response, ground effects and atmospheric conditions influence the ballast flight occurrence. By implementing laboratory wind tunnel tests and field-measuring tests, previous studies have built mechanical models of the ballast flight and simulated the process. However, rare existing reliability model is established to assess the probability of the ballast fly under the passage of the high-speed train. The purpose of this article is to present a reliability assessment framework for ballast flying phenomenon, which considers the force situation of the ballast particles and utilizes flow field measurements on commercial speeds. Reliability calculation process of this problem is conducted by the design point method which is easy to obtain with high calculation accuracy. By means of the reliability model of the ballast flight, the sensitivity analysis is proposed to evaluate the impact of the parameters on this ballast flight phenomenon. As a conclusion, it is possible to address a quantitative way to evaluate the risk of the ballast fly and obtain the level of security for the high speed line.**

Keywords - Ballast flight; High speed railway; Reliability; Risk assessment; Sensitivity analysis.

I. INTRODUCTION

Since the Tokaido Shinkansen High Speed Railway (HSR for short) in 1964, the speeds around the worldwide major railway lines truly increased step by step, and a continued success has been indicated in such countries where the lines were ultimately accomplished. Although the huge advantages of the high speed railway are evident, we also have to admit that because of the different features compared to the conventional railways, there are some adverse effects to traffic safety, such as the ballast-flying phenomenon appeared on the practical ballast-track line. In practice, the ballast flight injury like damaged wheel, broken glassed at station has been reported frequently and Fig. (1) illustrated. As a result, with the increase of the speed, we should attach importance to the reliability of the ballast, in order to guarantee the train's safety and durability. Some previous investigations about the reliability of ballasts have been operated and shown briefly below.

A semi-quantitative risk analysis model for the ballast flight was developed by Francesco Bedini Jacobini (2013), which defines the probability of the ballast flight, termed with the following formula[1],

$$R_{f_b} = P_d \times P_{fb|d} \times C \tag{1}$$

Where P_d is the probability that a ballast particle will displace from its rest position; $P_{fb|d}$ is the conditional probability that a ballast particle will fly given the displacement; and C is the consequence from the event of flying ballast. The article assessed the possible factors of the ballast flight risk, such as aerodynamic condition, track response, ground conditions, atmospheric conditions and presented a semi-quantitative risk matrix framework. This conceptual risk analysis cannot obtain the accurate likelihood of ballast flying and it lacks the concept of the reliability.

H. B. Kwon introduced the Ballast-Flying Probability Factor (BFPF) to estimate the risk of the ballast-flying quantitatively through the probability level [2]. In their research, the state of ballast under the strong wind and the mean wind speed in the full scale field should be recorded by the wind tunnel tests and the actual operating conditions, respectively. The BFPF can be computed by the following expressions,

$$
\begin{aligned}
&i)\ V_{track} < V_{\min}, \qquad BFPF = 0\\
&ii)\ V_{track} > V_{\max}, \qquad BFPF = 1\\
&iii) V_{\min} < V_{track} < V_{\max},\\
&BFPF = \int_{m_1}^{m_2} \frac{V_{track} - V_{\min}}{V_{\max} - V_{\min}} \frac{dm}{m_2 - m_1}
\end{aligned}
\tag{2}
$$

Where $V_{\min}$ and $V_{\max}$ represent the maximum and minimum wind velocity where the ballast particle displaces and V_{track} is the mean wind speed above the track. The probability of ballast-flying associated with train speed and ballast mass can be calculated through the aforementioned equations. However, this method just considers the influence of the mass and shape of ballast under high velocity of the ballast flying, ignoring some other significant factors, for example the interlock force among the particles. In addition, the risk calculated leaves the distribution of the probability out of consideration, so the failure probability is a counter-intuitive result based on probability reliability.

Then a method taking account of the reliability theory was presented by G. Saussine, using the Stress Strength Interference Analysis (SSIA)[3], which exploits probability distributions of strength and stress and graphs to identify the conditions under which a part will fail, as well as the probability of these conditions. The researchers confirm the stress by a parameter linking to the aerodynamic properties of train and the ballast motion. And characterize the strength based on the results of the numerical simulations.

The proposed relevant parameter for the stress part is the summation of signal power, which can be expressed as,

$$P_{Tot} = \frac{1}{t_2 - t_1} \sum_{i=1}^{N} \int_{t_1}^{t_2} \left\| V(\mathrm{t}) \right\|^2 dt \tag{3}$$

Based on the reliability theory, this computing approach allow to evaluate the failure probability established the foundation that the random variables are followed a normal Gaussian law; otherwise do not apply. Above all, significant errors may be introduced at increasing distances from linearizing point by neglecting higher order term, when the limit state function is nonlinear [4].

In conclusion, the purpose of this thesis is to develop the theory of the reliability assessment approach for ballast flying phenomenon by both the mechanical and probabilistic design.

Fig. (1). Ballast Flying Damage

The summarization of the paper is as below: Section 1 presents the introduction of the necessity to continue to conduct research for evaluating the reliability of the ballast flight; Section 2 introduces the ballast particle flying mechanism based on force equilibrium briefly, which is preparation for the reliability analysis next; Then section 3 explores the reliability analysis methodology to assess the risk of the ballast flying based on the worldwide research and reports; Moreover, the sensitivity analysis is addressed based on the reliability theory in order to make sure the effects of the factors on reliability index of the ballast flight in section 4; Finally, section 5 emphasizes the main conclusions of the present article.

II. MECHANISM OF BALLAST FLYING

According to Guoqing Jing's former literatures [5], the ballast begins to fly with at least two indispensable conditions: ballast particles and particle movement related to the train load inducing sleeper dynamic force and train inducing lateral wind. By high speed video recording of the ballast particles motion or ballast wind tunnel tests, part of the ballast flying mechanisms have been illustrated. The mechanics models were constructed based on the ballast particle force balance (Fig. (2) illustrated), and at the same time in consideration of the condition of the vibration and the wind effect. In vertical direction, the forces of the ballast particle are balanced by mg , F_i , F_w , F_a . Where mg is gravity force by mass, F_i is ballast particle interlock force, F_w is high speed train resulted wind force acted on ballast effective surface, and F_a is ballast acceleration force due to ballast bed vibration [5].

On the basis of d'Alembert principle, ballast particle equilibration is expressed as follow,

$$F_w + F_a = mg + ma_T + F_i \tag{4}$$

Ballast flying influence factors include the train dynamic effects of vibration and train winds, complicated multi-body dynamics coupled with wind fluid effects. As the ballast flying indication, it needs the ballast material factor and flying "conditions". From the ballast flying source, it is related with single ballast particle, but also railway ballast bed structure, such as gradation and geometry shape. Train dynamic force is a significant factor, both the track dynamic response and train

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aerodynamics effects. Based on the above ballast flying mechanics formulas, several ballast flying influence factors are analyzed, and then the corresponded engineering method or possible measures are proposed. The above characteristics of ballast flight mechanism of HSR theoretically could be used to investigate the reason and the factors, due to the simple method is presented, more work should be done to discover the risk assessment and determine the ballast stability states, for example, based on random process and risk assessment methods to conduct the risk assessment.

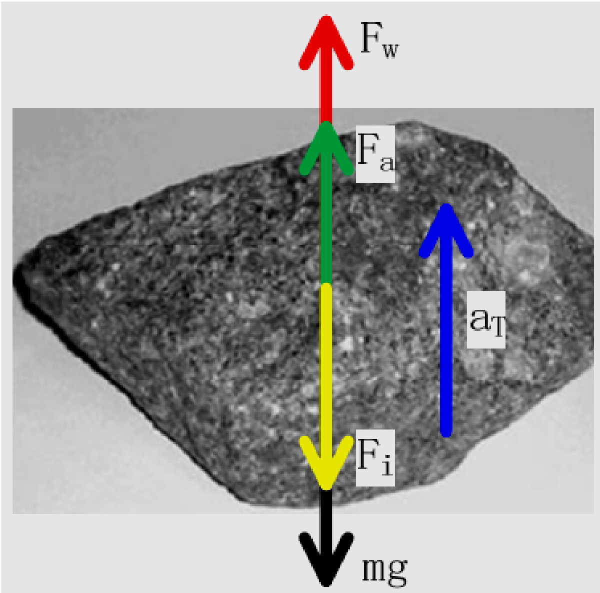

Fig. (2). Ballast Flying Mechanic Equilibrium.

III. RELIABILITY ANALYSIS OF BALLAST FLYING

Some necessary concepts need to be determined before the reliability analysis, such as the stress and strength proportion, the limit state and so on.

For the structural reliability, the limit state is the state of the structure including its loads at which the structure is just on the point of not satisfying the design requirements, it correspond to the maximum load capacity related to the formation of a mechanism in the structure, excessive plasticity, rupture due to fatigue and instability, etc[6].

No less than the structural reliability, the reliability analysis of ballast flying also should keep to the normal procedure. Whereas how to determine the reasonable stress and strength proportion for the ballast or what the limit state exactly is must be thought over.

A. Limit State

For engineering safety and simplified calculation consideration, assume the total acceleration a_T to be 0 and shift items. Then the formula (4) can be expressed as:

$$mg + F_i - F_w - F_a = 0 \qquad (5)$$

The function (5) can be considered as the critical state which indicates the particular ballast particle state of balance, and the corresponding train speed is the critical speed.

If $mg + F_i - F_w - F_a > 0$, it is demonstrated that the state of the ballast is steady and it will stay at its quondam position; If not, the ballast flying phenomena will occur.

Consequently for the ballast flying phenomenon, we propose to characterize the stress by the variability of the sum of the F_w, F_a and the strength using the results of $mg + F_i$.

Therefore, a limit state function of the ballast flight is defined,

$$g(\mathrm{X}) = R - S = mg + F_i - F_w - F_a \qquad (6)$$

In which X denotes the vector of stochastic variables which include the load variables ($F_w + F_a$) and the strength variables ($mg + F_i$).

B. Statistical Analysis

In the issue we talking about, four random variables' (m , F_i , F_w , F_a) probability distributions need to be ascertained from the data of the previous full scale field measurement. To be specific, some preliminary conclusions can be conveyed by these three expressions:

1) The distribution of the ballast's mass is earned from the particle size distribution and the density of the ballast approximately, and we presume the shape of the ballast is nearly spherical;

2) On account of the huge difficulty in measuring the dynamic ballast particle interlock force under the train, we presume it is equal to 0.1g, and in line with the preceding static force test, similar results can be got almost;

3) F_w and F_a have been measured and reported in the literature[7]. Because there is considerable difficulty in measuring the wind suction in the real condition, we have to replace the exact value by the approximation taking some wind engineering standards, textbooks, or famous literatures [8-12]. So F_w is computed by the product of the wind pressure over the ballast particle and a reduction coefficient considering the wind pressure distribution. Hence the approximate result can be achieved. Then the probability distribution analysis can be carried out and Fig. (3)/(4) shows the histogram of them together with the

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corresponding Normal and Extreme Value probability distribution function fit. As a result, we choose the Extreme Value type I for the distribution of F_w.

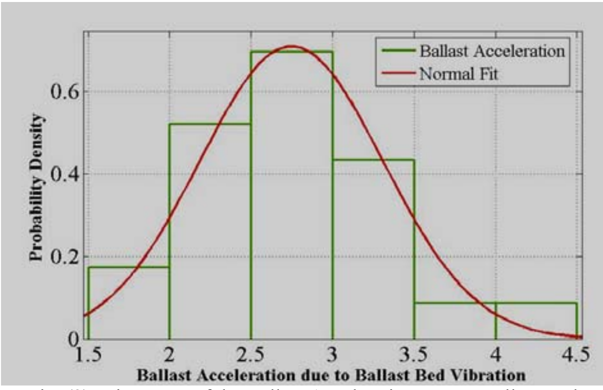

Fig. (3). Histogram of the Ballast Acceleration Due to Ballast Bed Vibration.

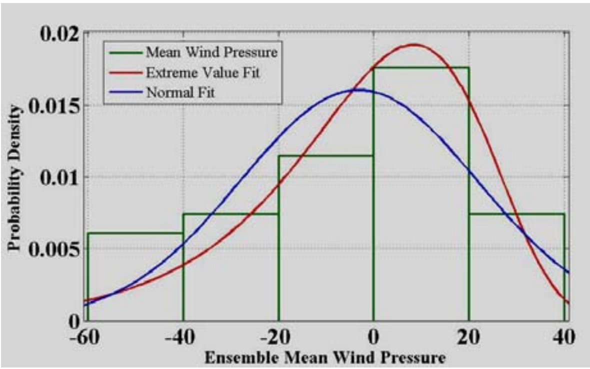

Fig. (4). Histogram of Wind Force Acted on Ballast Effective Surface.

In a word, the results of the statistical analysis are shown in the Table 1.

TABLE 1. STATISTICAL CHARACTERISTIC OF THE RANDOM VARIABLES

Variable	Distribution	Statistical Parameter
mg	Normal	$\mu = 6.200$, $\sigma = 0.100$
F_i	Constant	$\mu = 0.1g = 1.000$, $\sigma = 0$
F_w	Extreme Value I	$\mu = 3.119$, $\sigma = 0.0252$
F_a	Normal	$\mu = 1.674$, $\sigma = 0.563$

C. Reliability Index and Failure Probability

To overcome the disadvantages of the Mean value FOSH method, Hasofer, Lind, etc proposed the advanced method, named Check Point Method which has the following preponderances:

1) It can solve the problem that the random variables is not the normal distribution;

2) High computational accuracy with the less computing work relatively;

3) The check point value computed can be applied for designing expediently.

The first step in reliability-based assessment is the estimation of failure probability, which is computed as,

$$P_f = \int_{g(X)<0} f_X(x)\,dx \tag{7}$$

In which $f_X(x)$ is the probability distribution of x, it means the random variable.

For the convenience of expression, now change the equation (6) into,

$$g(X) = X_1 + X_2 - X_3 - X_4 \tag{8}$$

Where X_1, X_2, X_3, X_4 represent mg, F_i, F_w, F_a, respectively.

And then the variables X_i are transformed to reduce variables with zero mean and unit variance through:

$$x_i = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{9}$$

In the space of reduced coordinates x_i, the limit state is the function (10), with failure occurs when $g_1 < 0$.

$$g_1(x_1, x_2, x_3, x_4) = 0 \tag{10}$$

Now we can define a reliability index β as the shortest distance between the surface $g_1 = 0$ and the origin. The point $(x_1^*, x_2^*, x_3^*, x_4^*)$ on g_1=0 which corresponds to the shortest distance is referred to as the checking point and can be determined by solving the system of equations, shown in Fig. (5),

$$\alpha_i = \frac{\partial g_1 / \partial x_i}{\left[\sum (\partial g_1 / \partial x_i)^2\right]^{1/2}} \tag{11}$$

$$x_i^* = -\alpha_i \beta \tag{12}$$

$$g_1(x_1^*, x_2^*, \ldots, x_n^*) = 0 \tag{13}$$

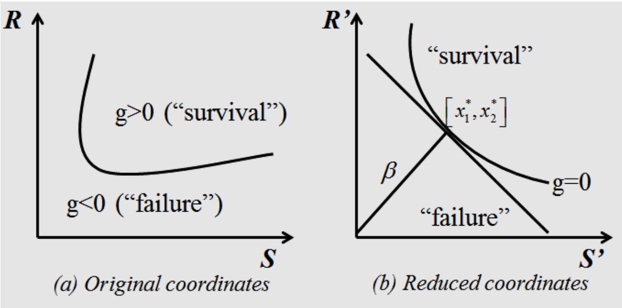

Fig. (5). Formulation of Safety Analysis in Original and Reduced Variable Coordinates

Correspondingly, the failure probability has the relationship with the reliability index,

$$P_f = \Phi(-\beta) \tag{14}$$

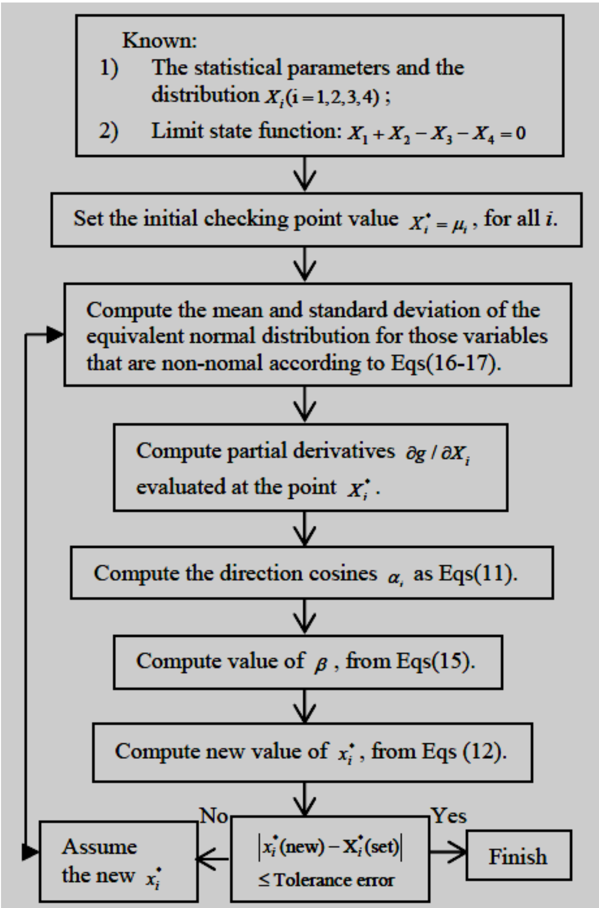

Fig. (6). Iteration Computing Process of the Reliability Index.

Since a certain random (F_w) variable of the ballast flying is regarded as the Extreme Value distribution, the reliability index can be expressed as the formula (12) through the equivalent transformation,

$$\beta = \frac{\mu_{x_1} + \mu_{x_2} - \mu'_{x_3} - \mu_{x_4}}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2 + \sigma_{x_3}'^2 + \sigma_{x_4}^2}} \tag{15}$$

$$\mu'_{X_3} = \frac{x_3^* - \Phi^{-1}\left[F_{X_3}(x_3^*)\right]}{\sigma_{X_3}^*} \tag{16}$$

$$\sigma_{X_3}^* = \frac{\varphi\left\{\Phi^{-1}\left[F_{X_3}(x_3^*)\right]\right\}}{f_{X_3}(x_3^*)} \tag{17}$$

In which, μ'_{X_3} , $\sigma_{X_3}^*$ are the equivalent mean value and the mean root square of the random variable(F_w), respectively; $\Phi(\cdot)$ represents the standard normal distribution; $\Phi^{-1}(\cdot)$ means the inverse function of standard normal distribution; $\varphi(\cdot)$ is the probability density of standard normal distribution; F_{X_3}, f_{X_3} are the distribution function and the probability density function of the F_w , respectively; x_3^* is the check point of the F_w .

According to the form descriptions, μ'_{X_3} , $\sigma_{X_3}^*$ are obtain by the value of the check point x_3^* , which we haven’t know already. As a result, the iteration method should be implemented, whose computing process is shown in Fig. (6).

Finally, the reliability index for the ballast flying issue is 3.2757 , correspondingly whose failure probability is equal to 5.2699×10^{-4} .

IV. SENSITIVITY ANALYSIS

Based on the preceding part of the paper, the sensitivity analysis can be carried out through the reliability theory, aiming to estimate the influencing factors’ impact to the ballasting flying.

First, the basic random variables X_i should be transformed into standard normal random variables Y_i . So the limit state function can be expressed as $G(\mathrm{Y})$. Then take the Taylor series expansion at the check point[13],

$$G(\mathrm{Y}) \approx \mathrm{G}(\mathrm{Y}^*) + \sum_{i=1}^{n} \frac{\partial \mathrm{G}(\mathrm{Y}^*)}{\partial Y_i}(Y_i - Y_i^*) \tag{18}$$

Assume the vector quantity of the parameters of the distribution is $d = (\mathrm{d}_1, \mathrm{d}_2, ..., \mathrm{d}_k)^T$, and because of $\beta = \sqrt{Y^{*\mathrm{T}} Y^*}$, the equation $\frac{\partial \beta}{\partial d_i} = \frac{1}{2}\left(Y^{*\mathrm{T}} Y^*\right)^{-\frac{1}{2}} \frac{\partial (Y^{*\mathrm{T}} Y^*)}{\partial d_i}$ is workable.

Considering the relationship between the reliability index β and the check point Y^*, the following expression can be obtained,

$$\frac{\partial\beta}{\partial d_i}=\frac{1}{\beta}Y^{*\mathrm{T}}\frac{\partial}{d_i}(\mathrm{TX}^*+\mathrm{B}) \tag{19}$$

In which, T and B are conversion matrix of the standard normal distribution. Owing to

$$\frac{\partial}{\partial di}(\mathrm{TX}^*)\approx\frac{T_{d+\Delta d_i}\tilde{X}^*-T_d X^*}{\Delta d_i} \tag{20}$$

In which, $T_{d+\Delta d_i}$ *and* $\tilde{X}^*$ are conversion matrix and the check point with a small increment Δd_i, respectively. Thanks to Kramchandani [14] the following close approximation is set up,

$$\frac{T_{d+\Delta d_i}\tilde{X}^*-T_d X^*}{\Delta d_i}\approx\frac{T_{d+\Delta d_i}X^*-T_d X^*}{\Delta d_i} \tag{21}$$

Consequently,

$$\frac{\partial}{\partial d_i}(\mathrm{TX}^*)\approx\frac{T_{d+\Delta d_i}\tilde{X}^*-T_d X^*}{\Delta d_i}\approx\frac{\partial T}{\partial d_i}\mathrm{X}^* \tag{22}$$

And then,

$$\frac{\partial\beta}{\partial d_i}\approx-\frac{\left[\nabla G(\mathrm{Y}^*)\right]^T}{\left\|\nabla G(\mathrm{Y}^*)\right\|}\left(\frac{\partial T}{\partial d_i}X^*+\frac{\partial B}{\partial d_i}\right) \tag{23}$$

If the basic random variables are uncorrelated and follow Gaussian distribution, the formula can be derived below,

$$\frac{\partial\beta}{\partial\mu_{X_i}}=\frac{1}{\sigma_{X_i}}\frac{1}{\left\|\nabla G(\mathrm{Y}^*)\right\|}\frac{\partial G(\mathrm{Y}^*)}{\partial Y_i} \tag{24}$$

$$\frac{\partial\beta}{\partial\sigma_{X_i}}=\frac{Y_i^*}{\sigma_{X_i}}\frac{1}{\left\|\nabla G(\mathrm{Y}^*)\right\|}\frac{\partial G(\mathrm{Y}^*)}{\partial Y_i} \tag{25}$$

In order to identify the relative influence of the variables on the reliability index, the following equations are conducted for nondimensionalization, in which C_Y means the covariance matrix of Y.

$$\frac{\partial\beta}{\partial\mu_{X_i}}\sigma_{X_i}=\frac{1}{\left[\nabla G(\mathrm{Y}^*)^T C_Y\nabla G(\mathrm{Y}^*)\right]^{1/2}}\frac{\partial\beta}{\partial X_i}\sigma_{X_i} \tag{26}$$

$$\frac{\partial\beta}{\partial\sigma_{X_i}}\sigma_{X_i}=\frac{\partial\beta}{\partial\mu_{X_i}}\left(y_i^*-\mu_{X_i}\right) \tag{27}$$

Because X_3 doesn't satisfy the normal distribution, equivalent normalization should be taken first, and at same time X_2 are constant as an assumption, which is regarded as invariable.

As a result, the distribution parameters of the influencing factors' effect to the reliability index and the risk of the ballast flight can be achieved by the method above. After nondimensionalization, the final results are given in Table 2, we can find the X_4 which means the force induced by the vibration of the track makes a greater impact on the reliability index, seen in the Fig. (7).

TABLE 2. SENSITIVITY ANALYSIS OF RELIABILITY INDICES TO PARAMETERS

X	$\frac{\partial\beta}{\partial\mu_{X_i}}$	$\frac{\partial\beta}{\partial\mu_{X_i}}\sigma_{X_i}$	$\frac{\partial\beta}{\partial\sigma_{X_i}}$	$\frac{\partial\beta}{\partial\sigma_{X_i}}\sigma_{X_i}$
X_1	10.0	1.3	61.4	6.1
X_3	39.7	1.2	-46.1	-1.9
X_4	2.8	3.1	14.8	9.0

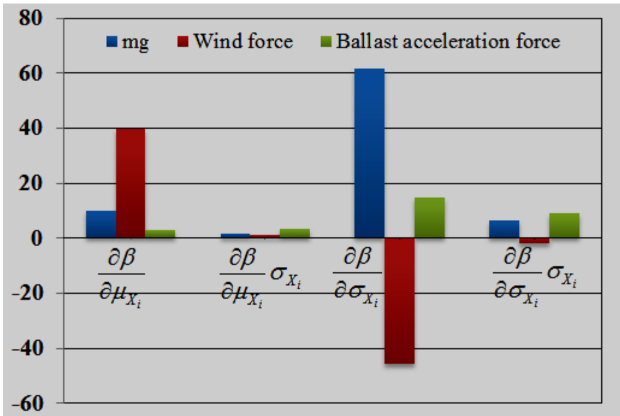

Fig. (7). Sensitivity Analysis of Reliability

V. CONCLUSIONS

In this article, we develop a general framework to obtain the reliability index of the ballast flight, which is on the strength of field measurement and the Check Point Method. By this approach, the safety level of the operation of the high speed line including the train and the track can be appraised quantitatively, so that our design will become more and more safe, economical and convenient. And we also conduct the approach of the sensitivity analysis to consider the distribution parameter of the influencing factors' contribution to the reliability index and the risk occurrence of ballast flying.

As the data have not been surveyed completely, the final value may be not so precise, but the concept of the method is reasonable and meaningful.

The upcoming program is to consummate the test data and carry out some simulation of the ballast flight phenomenon, which can compare with the calculated value by theory.

CONFLICT OF INTEREST

The authors confirm that this article content has no conflicts of interest.

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