MULTI-PHYSICS LATTICE DISCRETE PARTICLE MODEL (M-LDPM) FOR THE COUPLING OF DIFFUSION PROCESSES AND FRACTURE

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Carbon neutral: CO₂ emissions

- CO₂ emissions, are of primary resources of greenhouse gas emissions, influence the global climate change.
- Construction industry, is responsible for up to 10% of total CO₂ emissions per year (ACI, 2018).
- According to the *Paris Agreement*, the major carbon emitters need to cut the emission to limit global warming "well below" 1.5 °C (2.7 °F) of current level.



Figure 1: CO₂ emissions and global climate change ^[1]

Carbon neutral: durability & sustainability

- A key factor which will lessen the environmental footprint of building materials is improving the durability and sustainability.
- A comprehensive understanding of the <u>multiphysical phenomena</u> will be vital to ensure an optimal life-cycle of the structure, and the minimization of environmental impacts.



Figure 2: Concrete chloride attack ^[2]



Figure 3: Crumbing concrete driveway^[3]

 [2] Credit: https://www.giatecscientific.com/education/service-life-prediction-forreinforced-concrete-exposed-to-chloride-induced-corrosion-risk/
 [3] Credit: https://gfpcement.com/correcting-concrete-3-signs-need-repair-work

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Multiphysics-LDPM framework

• The Lattice Discrete Particle Model (LDPM) has proved its efficiency on simulating softening and fracture of quasi-brittle materials such as concrete, shale, etc., while the Flow Lattice Model (FLM), a topologically dual lattice model of LDPM, has been proposed for diffusion/flow problems.



Figure 4: LDPM-FLM coupling framework setup: a) LDPM discretization, b) FLM network (adopted from [4])

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^[4] Image credit: Shen, Lei, et al. "Multiphysics lattice discrete particle model for the simulation of concrete thermal spalling." *Cement and Concrete Composites* 106 (2020): 103457.

Multiphysics-LDPM framework

• The Lattice Discrete Particle Model (LDPM) has proved its capability on simulating softening and fracture of quasi-brittle materials such as concrete, shale, etc., while the Flow Lattice Model (FLM), a topologically dual lattice model of LDPM, has been proposed for diffusion/flow problems.



Figure 5: LDPM and the *Flow Lattice Model* setup: a) concrete mesostructure and LDPM tessellation in 2D, b) conduit Flow Lattice element in association with adjacent LDPM tetrahedra in 3D

Flow lattice element formulation – saturated flow

• Balance equation in each control volume V_I associated with node N_I ^[5]:

 $V_I \mathcal{C}(p_I)\dot{p}_I + Aj = V_I \mathcal{S}(p_I) \qquad I \in 1,2 \quad (1)$

Fick's first law of diffusion governs the diffusion flux density:

$$j = -\xi(p)\frac{\partial p}{\partial x} \tag{2}$$

The discrete estimation of gradient between N_1 and N_2 reads:

$$\frac{\partial p}{\partial x} = \frac{\Delta p}{l} \mathbf{e} = \frac{p_1 - p_2}{l} \mathbf{e}$$
(3)

The discretized balance equation for flow lattice element:

$$\begin{cases} V_{1}C(p_{1})\dot{p}_{1} - A\xi(\bar{p})\frac{p_{2} - p_{1}}{l} = V_{1}S(p_{1}) & (4a) \\ V_{2}C(p_{2})\dot{p}_{2} + A\xi(\bar{p})\frac{p_{2} - p_{1}}{l} = V_{2}S(p_{2}) & (4b) \\ & &$$

 $A = A_0 \mathbf{e} \cdot \mathbf{n}$

- *l*-FLE length
- C capacity
- S source/sink term
- ξ permeability



Figure 6: Flow Lattice Element (FLE) geometry in 3D

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^[5] Credit: Li, W., Zhou, X., Carey, J.W., Frash, L.P. and Cusatis, G., 2018. Multiphysics lattice discrete particle modeling (M-LDPM) for the simulation of shale fracture permeability. *Rock Mechanics and Rock Engineering*, 51, pp.3963-3981.

Coupled fracture-flow analysis

• Coupled fracture-flow governing equation in the FLM ^[6]:

$$\mathbf{u} = \begin{bmatrix} p_1 & p_2 \end{bmatrix}^{\mathrm{T}} \qquad \mathbf{f} = \mathbf{M}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} - \mathbf{S} = \mathbf{0}$$

$$\begin{bmatrix} V_1C_1 & 0\\ 0 & V_2C_2 \end{bmatrix} \dot{\mathbf{u}} + \frac{A}{l} \begin{bmatrix} \xi & -\xi\\ -\xi & \xi \end{bmatrix} \mathbf{u} - \begin{bmatrix} V_1S_1\\ V_2S_2 \end{bmatrix} = \mathbf{0}$$
where $C_i = M_b^{-1} + V_{ci} (K_f V_i)^{-1}$

$$\xi = \frac{\bar{p}_f (\kappa_0 + \kappa_c)}{\rho_{f0}\mu_f}$$

$$\kappa_c = \frac{1}{12A} \left(\frac{g_2}{I_{c1}} + \frac{g_1}{I_{c2}}\right)^{-1} I_{ci} = \sum_{j=1}^3 l_{fj} (\delta_{Nj}^i)^3$$

$$S_i = b\dot{\varepsilon}_{Vi} + \rho_{i}\dot{V} \left(\rho_{f0}V_i\right)^{-1}$$

$$\dot{v}_{ci} = \frac{V_{ci_{t+\Delta t}} - v_{ci_t}}{\Delta t}$$
 rate of volumetric strain

 p_i - nodal pore pressure (i = 1,2)

- V_i uncracked control volume
- M_b Biot modulus of the porous media

 K_f - fluid bulk modulus

 $ar{
ho}_f\,$ - average fluid density

 κ_0 - intrinsic permeability of the porous media κ_c - permeability of the cracked volume according to 2D Poiseuille flow μ_f - fluid viscosity

 $\dot{\epsilon}_{Vi}$ - rate of volumetric strain

b - Biot coefficient



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^[6] Ref: Rice, James R., and Michael P. Cleary. "Some basic stress diffusion solutions for fluid-saturated elastic porous media with compressible constituents." *Reviews of Geophysics* 14.2 (1976): 227-241.

Multiphysics problems in LDPM-FLM framework

• Different meshes, different time scales of the coupled-fields complicate the coupling process (a.k.a. "multidomain" or "multimodel" coupling).



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^[7] Credit: Li, Weixin. Computational and experimental characterization of the behaviors of anisotropic quasi-brittle materials: Shale and textile composites. Diss. Northwestern University, 2018.



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Two-way coupling between solvers

- The LDPM-FLM coupling framework uses the Inter-process communication (IPC) tools for the data-exchange between solvers.
 - For UNIX-based systems (Linux, NU Quest) named pipes
- Coupling scheme:



Parallel Coupling Scheme Figure 10: Parallel coupling scheme used in the LDPM-FLM framework

Time incrementation scheme:



Subcycling Time Incrementation Scheme

Figure 11: Time incrementation scheme used in the LDPM-FLM framework

• In Abaqus implementations, the algorithms were embedded in Fortran user subroutines.

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• Benchmark 1: poroelasticity problem, 1D Terzaghi's consolidation.



Figure 12: Setup for the two-way coupled, 1D Terzaghi's consolidation problem

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^[8] Ref: Detournay, Emmanuel, and Alexander H-D. Cheng. "Fundamentals of poroelasticity." *Analysis* and design methods. pergamon, 1993. 113-171.

• Benchmark 2: poroelasticity problem, radial expansion in a thick-walled hollow cylinder due to fluid injection.



Figure 14: Setup for the one-way coupled, poroelastic radial expansion problem

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 Benchmark 2: poroelasticity problem, radial expansion in a thick-walled hollow cylinder due to fluid injection.



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^[9] Ref: Grassl, Peter, et al. "On a 2D hydro-mechanical lattice approach for modelling hydraulic fracture." *Journal of the Mechanics and Physics of Solids* 75 (2015): 104-118.

• Benchmark 3: hydraulic fracturing of hollow thick-walled cylinder due to fluid injection.



Figure 16: Setup for the two-way coupled, hydraulic fracturing problem

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• Benchmark 3: hydraulic fracturing of hollow thick-walled cylinder due to fluid injection.



dimensionless radial displacement at the inner boundary of the hollow cylinder

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• Benchmark 3: hydraulic fracturing of hollow thick-walled cylinder due to fluid injection.



Figure 18: a) Crack patterns (crack opening contours) and b) pressure contours for uncoupled condition at three moments marked in Fig. 37

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• Benchmark 3: hydraulic fracturing of hollow thick-walled cylinder due to fluid injection.



Figure 19: a) Crack patterns (crack opening contours) and b) pressure contours for fullycoupled condition (b = 1.0) at three moments marked in Fig. 37

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Summary

- A multiphysics framework for Lattice Discrete Particle Model (LDPM)-Flow Lattice Model (FLM) coupling has been developed.
- The multiphysics framework is capable to solve poroflow (poroelasticity, hydraulic fracturing) problems accurately.
- The coupled analysis shows the effects of Biot's coefficients on the crack pattern, as well as the pressure diffusion in hydraulic fracturing.

Suggested work

- Extend the multiphysics framework for the coupling with more physical fields (e.g., temperature, chemical, biochemical components).
- Incorporate the parallel computing in the multiphysics framework to improve the efficiency.

Questions?



Topologically dual lattices

• The topological duality (e.g., Voronoi-Delaunay duality), has been brought to describe many coupled physical phenomena, such as aligned cracks and conduit elements allowing to accurately reflect the crack opening effect on the flow.



Figure 5: Voronoi-Delaunay duality ^[5]



Figure 6: A 2D dual lattices using the concept of Voronoi-Delaunay duality ^[6]

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^[5] Credit: https://mathworld.wolfram.com/DelaunayTriangulation.html.

^[6] Credit: Hwang, Young Kwang, et al. "Compatible coupling of discrete elements and finite elements using Delaunay–Voronoi dual tessellations." *Computational Particle Mechanics* 9.6 (2022): 1351-1365.

Application: hygro-thermal-chemical evolution in fresh concrete

The discrete implementation of the HTC model (Di Luzio and Cusatis 2009):

(9d)

$$W_{I} \frac{\partial W_{e}}{\partial H} \dot{H}_{I} + S^{*} j_{H} = W_{I} q_{H}$$

$$W_{I} \rho c_{T} \dot{T}_{I} + S^{*} j_{T} = W_{I} q_{T}$$

$$I \in P, Q$$
(6b)

The moisture and heat flux density are governed by an equivalent Darcy's law and Fourier's law, respectively:

$$j_H = -D_H(H,T)\frac{\partial H}{\partial x}$$
 (7) $j_T = -\kappa \frac{\partial T}{\partial x}$ (8)

Hygro-Thermo-Chemical

- w_{e} evaporable water content
- ρ concrete density
- c_{T} specific heat of concrete
- S^* area associated with *j*
- D_{H} moisture permeability
- κ heat conductivity
- q_H moisture source/sink term
- q_T heat source/sink term

$$\begin{cases} W_{P} \frac{\partial W_{e}}{\partial H} (H_{P}, T_{P}) \dot{H}_{P} - S^{*} D_{H}(\overline{H}, \overline{T}) \frac{H_{Q} - H_{P}}{l} = W_{P} q_{H}(H_{P}, T_{P}) \quad (9a) \\ W_{P} \rho c_{T} \dot{T}_{P} - S^{*} \kappa \frac{T_{Q} - T_{P}}{l} = W_{P} q_{T}(H_{P}, T_{P}) \quad (9b) \\ W_{Q} \frac{\partial W_{e}}{\partial H} (H_{Q}, T_{Q}) \dot{H}_{Q} + S^{*} D_{H}(\overline{H}, \overline{T}) \frac{H_{Q} - H_{P}}{l} = W_{Q} q_{H} (H_{Q}, T_{Q}) \quad (9c) \\ W_{Q} \rho c_{T} \dot{T}_{Q} + S^{*} \kappa \frac{T_{Q} - T_{P}}{l} = W_{Q} q_{T} (H_{Q}, T_{Q}) \quad (9d) \end{cases}$$

Let
$$C_P = \frac{\partial w_e}{\partial H}(H_P, T_P), C_T = \rho c_T, C_Q = \frac{\partial w_e}{\partial H}(H_Q, T_Q),$$

 $\overline{D}_H = D_H(\overline{H}, \overline{T}), q_{HI} = q_H(H_I, T_I), q_{TI} = q_T(H_I, T_I):$
 $\mathbf{u} = [H_P T_P H_Q T_Q]^T$ $\mathbf{f} = \mathbf{M}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} - \mathbf{S} = \mathbf{0}$

$$\begin{bmatrix} W_P C_P & 0 & 0 & 0 \\ 0 & W_P C_T & 0 & 0 \\ 0 & 0 & W_Q C_Q & 0 \\ 0 & 0 & 0 & W_Q C_T \end{bmatrix} \dot{\mathbf{u}} + \frac{S^*}{l} \begin{bmatrix} \overline{D}_H & 0 & -\overline{D}_H & 0 \\ 0 & \kappa & 0 & -\kappa \\ -\overline{D}_H & 0 & \overline{D}_H & 0 \\ 0 & -\kappa & 0 & \kappa \end{bmatrix} \mathbf{u} - \begin{bmatrix} W_P q_{HP} \\ W_P q_{TP} \\ W_Q q_{HQ} \\ W_Q q_{TQ} \end{bmatrix} = \mathbf{0}$$

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Coupling between solvers



Coupling between solvers



PhD Thesis Defense Presentation

Coupling between solvers



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PhD Thesis Defense Presentation

FLM Application: hygro-thermo-chemical evolution in fresh concrete

• Governing equation for HTC problem (Di Luzio and Cusatis 2009 paper ^[2] or <u>detailed derivation</u>):

$$V_{1} \begin{pmatrix} \frac{\partial w_{e}}{\partial h}\dot{h} + \frac{\partial w_{e}}{\partial T}\dot{T} + \frac{\partial w_{e}}{\partial \alpha_{c}}\dot{\alpha}_{c} + \frac{\partial w_{e}}{\partial \alpha_{s}}\dot{\alpha}_{s} + \dot{w}_{n} \end{pmatrix} + AD_{h}\frac{h_{2} - h_{1}}{l}\mathbf{e}\cdot\mathbf{n} = 0$$

$$\mathbf{f} = \mathbf{M}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} - \mathbf{S} = \mathbf{0} \qquad \mathbf{u} \quad \overline{T}_{2} [\underline{h}_{T_{1}}T_{1} \quad h_{2} \quad T_{2}]^{T}$$

$$V_{1}(\rho c_{t}\dot{T} + \dot{\alpha}_{s}s\tilde{Q}_{s}^{\infty} + \dot{\alpha}_{c}c\tilde{Q}_{c}^{\infty}) + A\lambda \frac{\overline{T}_{2}[h_{T_{1}}T_{1} \quad h_{2} \quad T_{2}]^{T}}{\mathbf{e}\cdot\mathbf{n}} = 0$$

$$V_{w} \begin{bmatrix} g_{1}C_{1} & g_{1}C_{2} & 0 & 0 \\ g_{1}C_{3}V_{2}g_{1}\frac{\partial w_{e}}{\partial h} & g_{2}C_{1}\frac{\partial \overline{T}_{2}}{\partial \overline{T}_{2}C_{2}} \\ 0 & 0 & g_{2}C_{3} & g_{2}C_{4} \end{bmatrix} \xrightarrow{\partial w_{e}A_{w}} \begin{bmatrix} h_{2}\frac{\partial w_{e}}{\partial \alpha_{c}}\dot{t} \\ h_{2}\frac{\partial w_{e}A_{w}}{\partial \alpha_{c}}\dot{t} \\ h_{2}\frac{\partial w_{e}}}{\partial \theta_{c}}\dot{t} \\ h_{2}\frac{\partial w_{e}}{\partial \theta_{c}}\dot{t} \\$$

Use
$$-\mathbf{f}(\mathbf{u}_n) = -(\mathbf{M}\dot{\mathbf{u}}_n + \mathbf{K}\mathbf{u}_n - \mathbf{S})$$
 as RHS
 $\frac{\partial \mathbf{f}(\mathbf{u}_n)}{\partial \mathbf{u}}$ as tangent stiffness (AMATRX)

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^[2] Credit: Di Luzio, G. and Cusatis, G., 2009. Hygro-thermo-chemical modeling of high performance concrete. I: Theory. *Cement and Concrete composites, 31*(5), pp.301-308.

Two-way coupling between solvers

- The mechanical analysis is done in Abaqus/Explicit, implemented with the Abaqus user-defined element VUEL, the transport analysis is done in Abaqus/Standard, implemented with Abaqus user-defined element UEL.
- The core functionality sequential coupling between two Abaqus solvers is achieved through data communication interface in FORTRAN subroutines.

```
С
      Two-way coupling processes
      call VGETOUTDIR (OUTDIR, LENOUTDIR)
                                             ! Work directory
      if (kstep -= 0) then ! Abagus/Explicit Packager stage
        ! if (Lop /= 0) then ! Second call of VUEL subroutine in Abaqus/Explicit Packager stage
            ! continue
        l end if
        continue
      else ! Abaqus/Explicit Analysis stage
        if (kinc == 0) then ! Initial data exchange settings
            if (tetID == nomaxel) then ! Exchange when loop to the last element
              ! Sending LDPM info to Abaqus/Standard solver
               open (V2U, file='LDPM2FLM.pipe', defaultfile=trim (OUTDIR), form='formatted',
                    status='old',action='write',access='stream')
              write(*,*) 'Sending LDPM analysis settings'
              write(V2U, '(I8)') nomaxel
              write(V2U, '(ES24.17)') period
               flush(V2U)
               close (V2U)
               ! Receiving FLM info from Abaqus/Standard solver
               open (U2V, file='FLM2LDPM.pipe', defaultfile=trim (OUTDIR), form='formatted',
                    status='old',action='read',access='stream')
               write(*,*) 'Retriving FLM analysis settings'
               read(U2V, '(I8)') nnode FLM
               write(*,*) "nnode FLM", nnode_FLM
               read(U2V, '(I8)') MPtypeFLM
               write(*,*) "MPtypeFLM", MPtypeFLM
              close (U2V)
              if (MPtypeFLM == 1) then
                nfieldFLM = 2
               else if (MPtypeFLM == 2) then
                nfieldFLM = 1
               end if
               if (allocated (FLM2LDPM DATA) --- 0) then
                allocate (FLM2LDPM DATA (nnode FLM, nfieldFLM), LDPM2FLM DATA (nomaxel, 13))
                allocate (FLM2LDPM DATA old (nnode FLM, nfieldFLM) )
                LDPM2FLM DATA = 0.0d0
                FLM2LDPM DATA = 0.0d0
                FLM2LDPM DATA old = 0.0d0
               end if
            end if
```

Figure 12: Data communication interface in FORTRAN subroutines

FLM Application: hygro-thermo-chemical evolution in fresh concrete

• Relative humidity and temperature evolution in a newly-constructed concrete dam



Figure 3: a) humidity and temperature profiles alo

and boundary conditions, c) evolution of humidity, temperature and cement hydration degree at points A, B, and C (Compared with the homogenized model in Eliáš et al. 2022 ^[3])

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^[3] Credit: Eliáš, Jan, Hao Yin, and Gianluca Cusatis. "Homogenization of discrete diffusion models by asymptotic expansion." *International Journal for Numerical and Analytical Methods in Geomechanics* 46.16 (2022): 3052-3073.